TEMPERATURE FIELD OF A POROUS BODY IN THE PRESENCE OF EVAPORATIVE COOLING

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The process of heat propagation in a porous solid for evaporative cooling of a plate, a cylinder, and a sphere is analyzed.

The question of the analytic calculation of the process of porous cooling has still not been sufficiently explored in the technical literature. Only problems of plates and cylinders [1-7] with fixed temperatures at the cold and hot surfaces have been examined.

Here the temperature distribution in a solid porous wall with a flow of liquid or gas to the hot surface is analytically investigated for three different bodies – an infinite plate, a thin-walled cylindrical tube, and a thin-walled hollow sphere. It is assumed that in the two latter cases the cooling component flows uniformly in the direction from the axis of the cylinder and the center of sphere to the walls. We shall adopt the mechanism of interaction between the skeleton of the solid and the fluid proposed in [1]. The real capillary structure of the solid is replaced by an equivalent system consisting of uniform parallel cylindrical channels through which the cooling liquid or gas flows. The temperatures of skeleton and coolant are assumed to be identical at every point. It is also assumed that heat transfer within the specimen takes place by heat conduction in the skeleton and the coolant. The coefficients λ_s, λ_L and C_L and the density of the liquid are assumed constant.

We shall set up the differential equation describing the process in question by equating the quantity of heat accumulating in a element of volume of the body due to heat conduction with the quantity of heat that goes into changing the enthalpy of the liquid. For the thin-walled cylinder and sphere, the flux density j_m over the thickness of the wall is assumed constant. In the case of a one-dimensional temperature field and symmetrical steady-state heat transfer we obtain the following differential equation, which must be satisfied by the temperature of the porous solid:

$$\frac{d^2t}{d\eta^2}+\frac{1}{\eta}(\Gamma-\xi_w\eta)\frac{dt}{d\eta}-\frac{\Gamma}{\eta}\xi_w t=0. \tag{1}$$

In Eq. (1), the terms with the multiplier $\xi_w = j_m C_L/\lambda_{eff}$ characterize the change in enthalpy of the liquid, the other two terms determine the heat conductivity of the porous body.

For evaporative porous cooling the boundary conditions take the following form:

$$t=t_1 \quad \text{for} \quad \eta\equiv x=-\delta \qquad \text{(plate)},$$
$$t=t_1 \quad \text{for} \quad \eta\equiv r=r_1 \qquad \text{(cylinder and sphere)}, \tag{2}$$

$$\alpha\Delta t=\lambda_{eff}\frac{dt}{d\eta}+\rho j_m \quad \text{for} \quad \eta\equiv x=-\delta,\ \eta\equiv r=r_1, \tag{3}$$

where $\Delta t = t_s - t_2$; λ_f if the effective thermal conductivity of the solid and liquid phases occupying the volume of the body ($\lambda_{eff} = \lambda_s(1-P)+\lambda_L P$).

According to (3), the heat supplied to the hot surface of the wall is spent on warming up the body by heat conduction and on varporizing liquid. If we take the gas as the inert component transmitted through the porous wall, its thermal conductivity can be neglected when $\lambda_s \gg \lambda_L$. Then $\lambda_{eff} = \lambda_s(1-P)$. Furthermore, $\rho j_m = 0$.

On solving differential equation (1) with boundary conditions (2) and (3) for a plate, a cylinder, and a sphere, respectively, we obtain

$$t_{pl}=(\xi_w\lambda_{eff})^{-1}[\exp(\xi_w x)-$$
$$-\exp(-\xi_w\delta)](\alpha\Delta t-\rho j_m),\quad -\delta\leqslant x\leqslant 0, \tag{4}$$

$$t_{cyl}=t_1\exp[\xi_w(r-r_1)](1-\zeta^{-1}\xi_w I\exp\xi_w r_2)+$$
$$+(\lambda_{eff}\zeta)^{-1}\exp(\xi_w r)(\alpha\Delta t-\rho j_m)I,\quad r_1\leqslant r\leqslant r_2, \tag{5}$$

$$t_{\mathrm{sph}} = t_1 (1 - \zeta^{*-1} \xi_w I^* \exp \xi_w r_2) \exp [\xi_w (r - r_1)] + \\ + (\lambda_{\mathrm{eff}} \zeta^*)^{-1} (\alpha \Delta t - \rho j_m) I^* \exp (\xi_w r), \quad r_1 \leqslant r \leqslant r_2, \tag{6}$$

where

$$\zeta = \xi_w I_1 \exp (\xi_w r_2) + K_2, \quad \zeta^* = \xi_w I_1^* \exp (\xi_w r_2) + K_2^2,$$

$$I = \int_{r_1}^{r} [r \exp (\xi_w r)]^{-1} dr = E_i (-\xi_w r) - E_i (-\xi_w r_1),$$

$$I^* = \int_{r_1}^{r} [r^2 \exp (\xi_w r)]^{-1} dr = [r_1 \exp (\xi_w r_1)]^{-1} - [r \exp (\xi_w r)]^{-1} - \\ - \xi_w [E_i (-\xi_w r) - E_i (-\xi_w r_1)],$$

$$I_1 = \int_{r_1}^{r_2} [r \exp (\xi_w r)]^{-1} dr, \quad I_1^* = \int_{r_1}^{r_2} [r^2 \exp (\xi_w r)]^{-1} dr,$$

$E_i(\eta)$ is the integro-exponential function tabulated in [8].

In a number of practical cases it is necessary to calculate the temperature of the wall t_1 on the cold side analytically. Accordingly, we set up the heat balance for the regions $-\infty \leq x \leq -\delta$ (plate) and $0 \leq r \leq r_1$ (cylinder and sphere), as a result of which we obtain the following differential equation, which must be satisfied by the temperature of the incident flow:

$$\eta^{\Gamma} \frac{d^2 t_{\mathrm{L}}}{d \eta^2} + (\Gamma \eta^{\Gamma - 1} - \xi) \frac{d t_{\mathrm{L}}}{d \eta} = 0, \tag{7}$$

where $\xi = J C_{\mathrm{L}} / \lambda_{\mathrm{L}}$.

The values of J (for a plate $J_{\mathrm{pl}} = j_m$, for a cylinder $J_{\mathrm{cyl}} = j_m^* r_1$, and for a sphere $J_{\mathrm{sph}} = j_m^* r_1^2$) are characterized, respectively, by the flux density per unit area and per unit length and by the total mass flow rate of the cooling component. The parameter j_m^* is determined by the flow of liquid in the section for $r = r_1$.

Boundary conditions for (7) can be represented in the form

$$t_{\mathrm{L}} = t_0 \quad \text{for} \quad x = -\infty \quad \text{(plate)}, \tag{8}$$

$$t_{\mathrm{L}} = t_0 \quad \text{for} \quad r = 0 \quad \text{(cylinder and sphere)},$$

$$\lambda_{\mathrm{L}} \frac{d t_{\mathrm{L}}}{d \eta} = \lambda_{\mathrm{eff}} \frac{d t}{d \eta} \quad \text{for} \quad \eta \equiv x = -\delta, \ \eta \equiv r = 0. \tag{9}$$

The latter condition assumes equality of the heat fluxes at the phase interface, i.e., at the boundary of contact between the liquid and the free surface of the body.

The solution of (7) with boundary conditions (8) and (9) is:

$$t_{\mathrm{Lpl}} = (\alpha \Delta t - \rho j_m) (\lambda_{\mathrm{L}} \xi_n)^{-1} \exp [\xi_{\mathrm{pl}} (x + \delta) - \xi_w \delta] + t_0, \tag{10}$$

$$t_{\mathrm{Lcyl}} = A^{-1} (r / r_1)^{\xi_{\mathrm{cyl}}} [\lambda_{\mathrm{eff}} \xi_w t_1 (r_1 \zeta - \exp \xi_w \Delta r) + \\ + \alpha \Delta t - \rho j_m] + t_0, \tag{11}$$

$$t_{\mathrm{Lsph}} = (A^*)^{-1} \exp (-\xi_{\mathrm{sph}} / r) [\lambda_{\mathrm{eff}} \xi_w t_1 (r_1^2 \zeta^* - \\ - \exp \xi_w \Delta r) + \alpha \Delta t - \rho j_m] + t_0, \tag{12}$$

where

$$\xi_{\mathrm{pl}} = J_{\mathrm{pl}} C_{\mathrm{L}} / \lambda_{\mathrm{L}}, \ \xi_{\mathrm{cyl}} = J_{\mathrm{cyl}} C_{\mathrm{L}} / \lambda_{\mathrm{L}}, \ \xi_{\mathrm{sph}} = J_{\mathrm{sph}} C_{\mathrm{L}} / \lambda_{\mathrm{L}},$$

$$A = \lambda_{\mathrm{L}} \xi_{\mathrm{cyl}} \zeta, \ A^* = \exp (-\xi_{\mathrm{L}} / r_1) \lambda_{\mathrm{L}} \xi_{\mathrm{sph}} \zeta^*.$$

Taking into account that $t_L|_{x=-\delta} = t_1$ (plate), and $t_L|_{r=0} = t_1$ (cylinder and sphere), from (10)-(12) we find the temperature t_1 for the three bodies investigated:

$$t_{1\mathrm{pl}} = (\xi_{\mathrm{pl}}\lambda_L)^{-1}(\alpha\Delta t - \rho j_m)\exp(-\xi_w\delta) + t_0, \quad (13)$$

$$t_1\mathrm{cyl} = (\zeta\lambda_L\xi_{\mathrm{cyl}}t_0 + \alpha\Delta t - \rho j_m)[\xi_w\lambda_{\mathrm{eff}}\exp(\xi_w\Delta r)]^{-1}, \quad (14)$$

$$t_{1\mathrm{sph}} = (\zeta^*\lambda_L\xi_{\mathrm{sph}}t_0 + \alpha\Delta t - \rho j_m)[\xi_w\lambda_{\mathrm{eff}}\exp(\xi_w\Delta r)]^{-1}. \quad (15)$$

Thus, the value of t_1 in (4)-(6) can be determined from relations (13)-(15).

From an analysis of solutions (4)-(6) and (13)-(15) it follows that for large values of the heat capacity of the liquid ($C_L \to \infty$) the temperature of the plate, thin-walled cylinder, or sphere approaches the temperature of the cooling liquid (gas) t_0.

The temperature of the wall tends to t_0 if $j_m \to \infty$; however, in this case, as follows from boundary condition (3) and the solutions for t and t_1, an abrupt temperature change occurs at the hot surface of the body. When $\lambda_f \to \infty$, for the temperature of the wall we have

$$t_{\mathrm{pl}} = t_0 + \gamma, \quad t_{\mathrm{cyl}} = t_0\frac{r_1}{r_2} + \gamma, \quad t_{\mathrm{sph}} = t_0\left(\frac{r_1}{r_2}\right)^2 + \gamma,$$

where $\gamma = (\alpha\Delta t - \rho j_m)/C_L j_m$.

NOTATION

η – space coordinate; Γ – constant (for an infinite plate $\Gamma = 0$; $\eta \equiv x$; for an infinite cylinder $\Gamma = 1$, $\eta \equiv r$; for a sphere $\Gamma = 2$, $\eta \equiv r$); δ – thickness of plate; t_s, t_1, t_2 – respectively, temperature of surrounding medium and wall on cold and hot sides; t_0 – temperature of cooling liquid at an infinite distance from cold surface of plate, on axis of cylinder or at center of sphere; ρ – heat of vaporization; λ_s and λ_L – thermal conductivities of skeleton of solid and cooling liquid, respectively; P – porosity of body; r_1 and r_2 – inside and outside radii of cylinder and sphere, respectively; $K = 1/r$.

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